Pre-Calculus

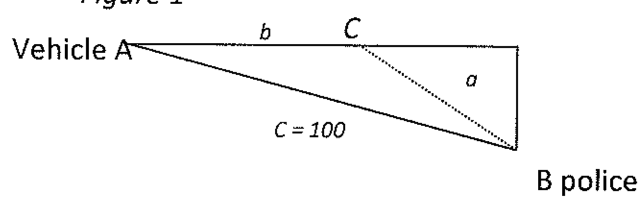
Project: Life in the Fast Lane Name: Rick Steffe

The life in the fast lane was an applied math problem that I was assigned to complete my Thursday afternoon. This problem is about how a police officer catches peoples speeds. With trigonometry and algebra we are able to find the true speed of the cars. We find there speed based upon the angle of the police officer from the car and the speed of the incoming car.

To measure speed accurately, police officers who use radar should be directly in front of the moving object. Obviously, that is not very practical, unless you want to have a very short career in law enforcement! In this project, we explore some factors that impact the accuracy of using radar to measure the speed of oncoming vehicles.

1. Suppose an officer in a car 15 feet off the side of the road (point B in Figure 1). A vehicle approaches traveling 70 mph (point A figure 1). We want to calculate the speed of the car reported by the radar unit when the car is 100 feet away.

Figure 1



a. We need to compute the speed of the car by measuring the difference in the length of BA and the length of BC. Generally, the length of AC (the true distance traveled by the car in *t* seconds) is not the same as the difference in the lengths of BA and BC (the distance the radar gun uses to compute the speed of the car). Where would the police officer need to be positioned for the distance the car travels and the distance the radar gun measures to be exactly the same?

Directly in front of the car.

b. We need to compute the length of BC as the first step in obtaining a function for computing the speed reported by the radar gun. To do so be use the formula

$$a = \sqrt{b^2 + c^2 - 2bc\cos A},$$

where a is the length of BC, b is the length of AC, and c is the length of BA.

i. Explain why equation (1) is valid.

It is valid because the law of cosine

ii. Find the *cosA*

CosA=√9775/100

iii. Determine an expression for *b* in terms of rate and time with the rate in feet per second and time as a variable in seconds. Remember that the car is traveling at 70 mph. 308/3t

See Calculations sheet

iv. Substitute your expression for *b* and *cosA* into the equation (1). You should now have a function where your input, *t*, is in seconds and your output, *a* is in feet.

See calculations sheet

$$a=\sqrt{\frac{308^2}{3t^2}+100^2-2\cdot 308/3\,t\sqrt{9775}}$$

c. The speed reported by the radar gun is

$$r_{rep}=\frac{100-a}{t}.$$

Find a function where your input, *t*, is in seconds and your output, r_{rep}, is in feet per second.

See calculations Sheet

d. Use the list of times given in the following table to compute various values of r_{rep}. Report your answers in both feet per second and miles per hour. Round to four decimal places.

Time (sec)	r_{rep} **(ft/sec)**	r_{rep} **(mph)**
0.1	102.6496	69.9884
0.01	102.0204	69.5593
0.0001	101.5102	69.2115
0.000001	101.5051	69.2081

2. In the previous table, the time becomes progressively smaller. Let's now assume the time is instantaneous. In this case, it can be shown (using calculus) that $r_{rep} = r_{car} cosA$. Use this formula to compute r_{rep} for question 1. How does this answer compare with the answers you obtained in part (d)?

$R_{rep} = 70 \cdot \frac{\sqrt{9775}}{112} = 69.2081$

For the rest of this project, use the formula
$r_{rep} = r_{car} cosA$ *to compute the speed reported by the radar gun.*

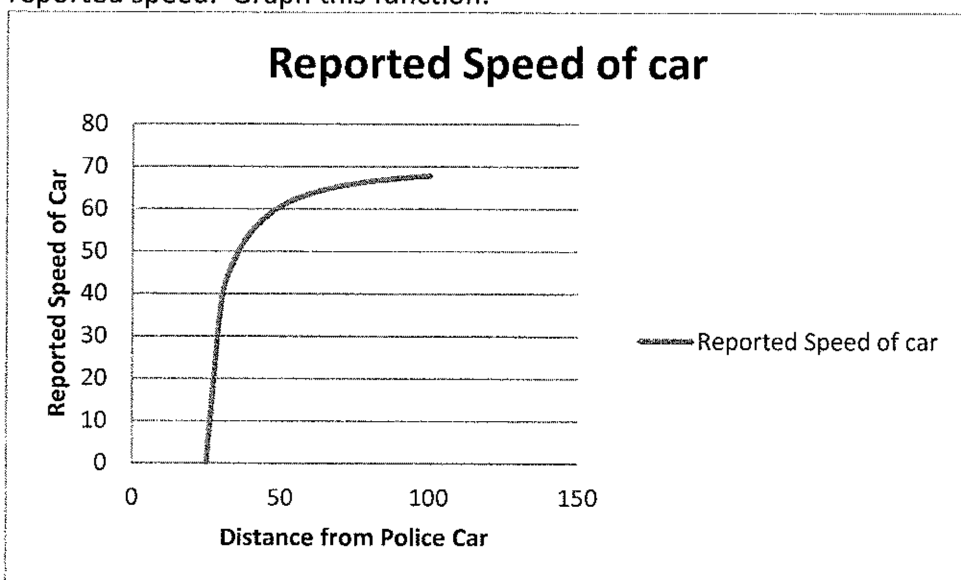
3. An officer is in a car 15 ft off the side of the road. A vehicle approaches traveling 70mph. What will the radar read as the car's speed when the car is 50 feet away?

$R_{rep} = 70 \cdot \frac{\sqrt{50^2 - 15^2}}{100} = 66.7$

4. Compare your answers in questions 2 and 3. Notice that the accuracy of the measurement (i.e. $|r_{car} - r_{rep}|$) changed as the vehicle came closer. Let's determine how the accuracy of the radar is affected by changing its distance from the vehicle, the speed of the vehicle, and the distance of the police officer from the side of the road.

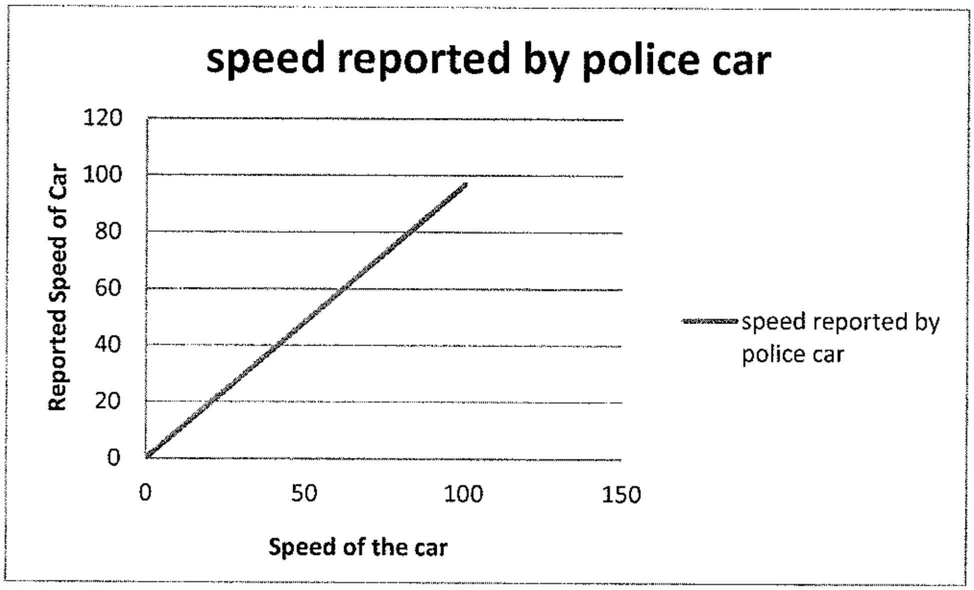
A- First, let's look at this problem graphically.

i. Holding speed of the car constant at 70 *mph* and distance from the police car to the side of the road constant at 25 ft, find a function where the input is the distance from the police car to the speeding car and the output is the reported speed. Graph this function.



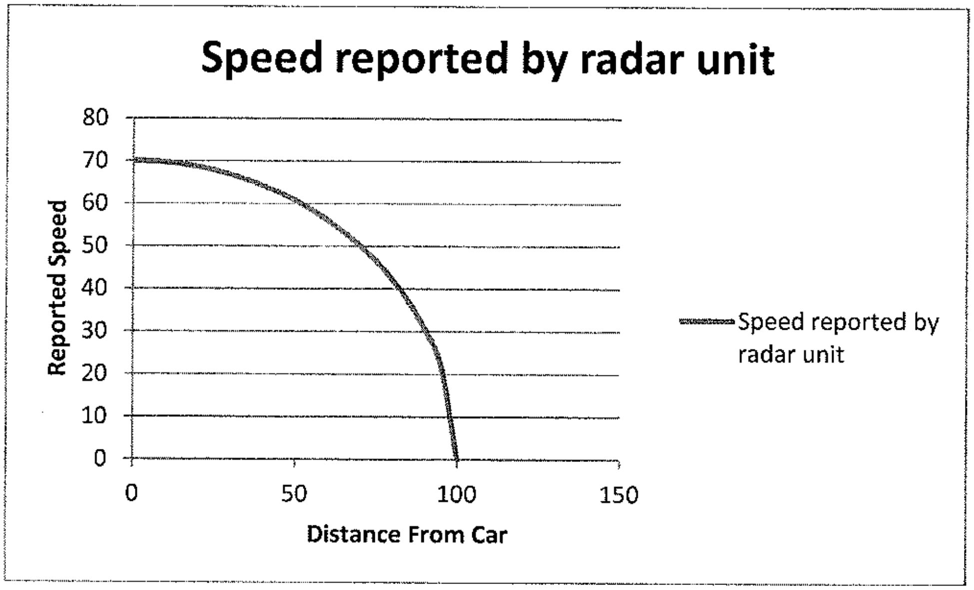
See calculations

Iii Holding distance from the speeding car constant at 100 ft and distance from the side of the road constant at 25 ft, find a function where the input is the speed of the car and the output is the speed reported by the radar unit. Graph this function.



See calculations

iii. Holding distance from the car constant at 100ft and speed of the car constant at 70mph, find a function where the input is the distance from the side of the road and the output is the speed reported by the radar unit. Graph this function.



See calculations

b. Using your graphs from part (a) as a guide, determine what happens to the accuracy of the radar reading as the vehicle comes closer. Determine what happens to the accuracy of the radar reading as the target vehicle goes faster. Determine what happens to the accuracy of the radar reading as the police officer sits farther off side of the road. Justify each of your three explanations with either a symbolic or a geometric argument.

As the vehicle comes closer the readings become much more accurate. When the car is farther away the readings are not as good.

5. In a realistic situation, people often first decide on how much error they are willing to have and then change the other variables accordingly. For example, the police officer may want the reading of the radar unit to be within 1 mph of the actual speed of the car. Assume the police officers are sitting in their car 10 feet off the side of an approaching car traveling 75 mph. How far away should the car be when the police officers use their radar unit to ensure that their reading is within the desired accuracy range?

See calculations

65 Ft away

6. A person who receives a speeding ticket for going 57mph in a 45 mph zone contests the ticket on the grounds that, because of the angle effect, his speed was actually less than what the radar indicated. As the court's expert witness (on the law of cosines), write a response to the judge indicating why you think the ticketed driver should or should not be fined for speeding.
This person deserves a ticket we discovered that the angle of the police officer and the car that is incoming. The significance is not that much. Most of the time it was about half of a mile per hour difference. This guy is speeding at 12 over 11 over isn't going to make a difference.

Applied Problem Calc.

bii $15^2 = 100^2 + \sqrt{9775} - 2(100)(\sqrt{9775}) \cos A$

$$\frac{\sqrt{9775}}{100} = \cos A$$

biii $70 \times 5280 = 369600/3600 = 102.6$ ft/sec $308/3t$ in numerator

biv $A = \sqrt{B^2 + C^2 - 2Bc \cos A}$

$a = \sqrt{\frac{308^2}{3t^2} + 100^2 - 2 \cdot 308/3t \cdot \sqrt{9775}}$

$a = \sqrt{\frac{308^2}{3t^2} + 100^2 - 2 \; 308/3t \; \frac{\cancel{100}}{\sqrt{9775}}}$

2) $R_{rep} = 70 \cdot \frac{\sqrt{9775}}{100} = 69.2081$

$R_{rep} = 70 \cdot \frac{\sqrt{50^2 - 15^2}}{100} = 66.7$

4ai $R_{rep} = 70\text{mph} \; \frac{\sqrt{x^2 - 25^2}}{x}$

4aii $R_{rep} = x \; \frac{\sqrt{100^2 - 25^2}}{100}$

4a iii $R_{rep} = 70 \; \frac{\sqrt{100^2 - x}}{100}$

5) $R_{rep} = 75 \cdot \frac{\sqrt{x^2 - 10^2}}{x}$

$75 \cdot x - 10$

$75 - 10 = 65$ ft away